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DISCRETE CHARACTER OF THE FORMATION OF VORTICES IN A DEVELOPING CIRCULATORY FLOW

V. S. Sadovskii and G. I. Taganov

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The problem of a developing circulatory flow past an airfoil impulsively brought from rest to a constant velocity in an inviscid, incompressible fluid was quantitatively solved for the first time (in the linear approximation) in the work of Wagner [1] more than 60 years ago. This work was based on the Prandtl's assumption of the continuous vortex shedding from the sharp trailing edge of an airfoil. The more general case of the unsteady problem of flow past a moving airfoil associated with the occurrence of flutter attracted, more than 50 years ago, the attention of Soviet scientists M. V. Keldysh, M. A. Lavrent'ev, A. I. Nekrasov, and L. I. Sedov, who developed the existing standard linear theory of the unsteady motion of an airfoil. The difficulties in the nonlinear problem, and their early concepts, are still valid today for the exact description of the flow and were elaborated by Sedov [2] 50 years ago.

Because of the difficulties in the exact description of the flow, we can replace the exact description of the vortex shedding from the trailing edge by a model. The problem of the developing circulatory flow past an airfoil impulsively brought from rest to a constant velocity was solved in [3, 4]. The flow past a flat plate with an angle of incidence $\alpha = 90°$ was examined by means of a dipole model under the assumption that the flow does not separate near the leading edge. It is emphasized that the dipole field represents not only the limiting case of a source–sink system but also the limiting case (in the direction perpendicular to the dipole axis) of a system of two vortices with velocity circulation of opposite signs. Therefore, the dipole model applied to describe flow containing domains with closed streamlines can be viewed as a degenerate classic Föppl's model with vortices of infinite circulation located on the surface of a body.

The above works show that, after approaching some critical instant of time t_*, the streamline which passes through the trailing edge no longer encloses the domain where trajectories of fluid elements form closed lines, and it was assumed that for $t > t_*$ this domain separates from the plate. Also, it was assumed that the new domain with closed trajectories of fluid elements was formed on the trailing edge after elapse of a period of time. The new domain grows until it reaches again a critical size at $t = t_{**}$, and so on.

Thus, the process of development of circulatory flow past an airfoil in an inviscid fluid characterized by vortex shedding from the trailing edge is not continuous but consists of subsequent formations and separations of domains with closed trajectories of fluid elements formed by discrete elements of the vortex sheet.

The supplementary information on the dipole model for $t = t_*$ presented below helps to estimate the circulation of the first vortex separated from the trailing edge of the plate as well as to examine the pattern of the flow for $t > t_*$ after bifurcation of a dipole.

The instantaneous patterns of flow for $t < t_*$ in the system of coordinates associated with the plate are shown in Fig. 1 for two descriptions of the flow. For an exact description the domain with closed trajectories of fluid elements is formed by the curled vortex sheet separating from the trailing edge of the plate (Fig. 1a). For the model description the analogous domain is formed by a dipole located at point D on the trailing edge of the

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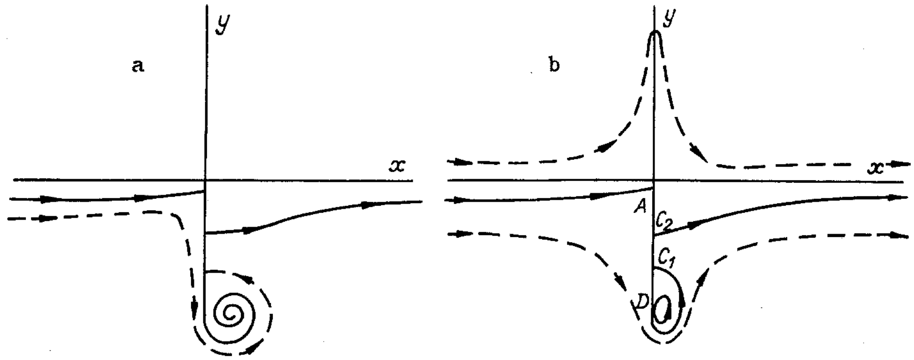

Fig. 1

plate (Fig. 1b). Simultaneously, the streamline which extends from the free stream to the critical point A and along the entire surface of the plate and detaches from the surface at point C_2 represents the limit of the dipole zone. The critical points A, C_1, and C_2 are located on the plate.

To examine the model flow as in [3, 4], we will assume quasi-steady character of the flow and we will use an auxiliary plane ζ. The domain of the flow is represented in the ζ-plane by the interior of a circle with unit radius. The location of the dipole with moment m is given on the circle by the angle β, and the dipole axis is directed along the diameter of the circle toward the point $\zeta = -i$. The complex potential w of the irrotational flow around the circle with the dipole and with unit velocity has at infinity the following form:

$$w(\zeta) = \zeta + \frac{1}{\zeta} - \frac{m}{2\pi} \frac{\exp\left[-i\left(\frac{\pi}{2} + \beta\right)\right]}{\zeta - \exp(-i\beta)}. \tag{1}$$

It follows from the condition $dw/d\zeta(-i) = 0$, which is necessary to ensure finite velocity at the sharp edge $z = -i$ of the plate, that in the physical plane $z = (1/2)(\zeta - 1/\zeta)$ we have

$$m = 8\pi(1 - \sin\beta) \equiv 8\pi d \tag{2}$$

(here d is the distance along the plate from the lower edge to the dipole).

The relation between the location of the critical points on the upstream and downstream sides of the plate and d was examined in [3, 4] for $0 < d \leqslant 0.098$. The critical points A and C at the initial instant of time $t = 0$ are located at the origin. For $t > 0$ the dipole moment $m > 0$ and the finite dipole zone with closed streamlines is formed on the downstream side of the plate. This zone is separated from the internal stream by the streamline which detaches from the sharp edge of the plate at $z = -i$ and reattaches to the plate at point C_2 on its downstream side. As a result, for $d > 0$ there are three critical points, A, C_1, and C_2, on the surface of the plate. Subsequently, the dipole moves upward along the plate, the size of the dipole zone increases, and the critical point A displaces upward on the plate. Simultaneously, points C_1 and C_2 move toward each other. For $t = t_*$ the dipole approaches location $d_* = 0.098$ ($\beta = 64.41°$), and the critical points C_1 and C_2 merge into one point located at a distance $s_* = 0.633$ from the lower edge.

This limiting pattern of the flow is shown in Fig. 2. It is shown below that the critical point C is moved away from the plate and is located in the stream while the dipole moves upward along the plate. Then, the flow pattern possesses different topological properties caused by bifurcation of the dipole. For the dipole model the presence of such qualitative rearrangement of the flow can be interpreted in the following way. The dipole zone with closed trajectories of fluid elements, formed prior to the instant of time $t = t_*$, is fully separated from the plate at $t > t_*$ and later drifts downstream.

Let us assume that the proposed dipole model of the formation of a domain with closed trajectories of fluid elements describes correctly the pattern with curled vortex sheet. Then, the critical dipole zone for $d_* = 0.098$ enables one to determine the circulation Γ_1 of the first vortex separated from the trailing edge and, consequently, the circulation $-\Gamma_1$ generated near the plate (adjoint vortex).

Let us assume that at the moment of separation of the dipole zone from the plate this domain contains fluid at rest (the dipole itself is needed only for the formation of the

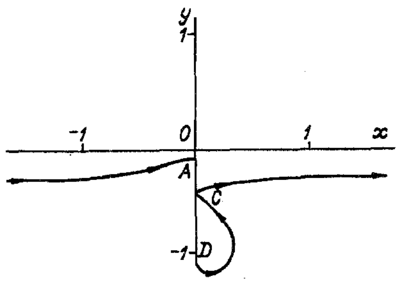

Fig. 2

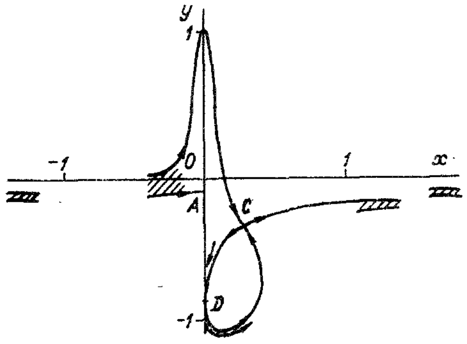

Fig. 3

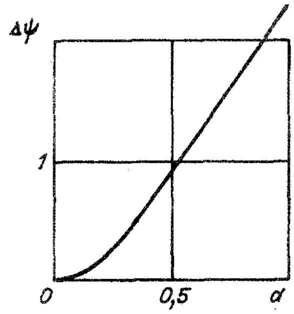

Fig. 4

TABLE 1

d	y_1	x_2	y_2
0,098	—0,072	0	—0,367
0,134	—0,094	0,297	—0,320
0,293	—0,188	0,667	—0,133
1,0	—0,544	1,115	0,772

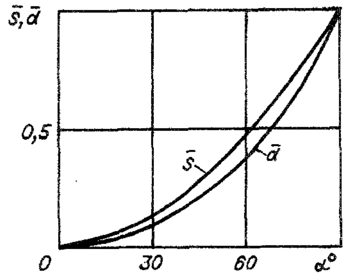

Fig. 5

zone). In this case, the liquid boundary of the domain becomes a vortical layer, the total strength (circulation) of which can be easily determined from Eq. (1) taking into account Eq. (2). A simple estimate shows that for a plate with chord equal to 2 and with unit velocity in the free stream, $\Gamma_1 = 1.612$ at infinity. Let us identify this quantity with the circulation of the first vortical domain (initial vortex) separated from the plate. Then, let us compare the same circulation with that of the stationary Zhukovskii flow past a plate with $\Gamma_\infty = 2$. Also, let this flow be stabilized over an infinite period of time since the trace consists of an infinite number of discrete vortex domains decreasing in size and moving downstream at an infinite distance. Then, we find $\Gamma_1 = 0.2566\Gamma_\infty$.

Consequently, for the plane characterizing the flow mass contained in the first vortical domain the calculation yields F = 0.149, and the average vorticity in the plane is $\omega \approx \Gamma_1/F = 10.82$.

Let the dipole be located at a distance $d > d_*$ from the lower edge of the plate which corresponds to its angular position $\beta < 64.41°$ on the circle in the ζ-plane. Using Eqs. (1) and (2) one obtains an algebraic equation of fourth order for the critical points. Obviously, one of its roots is $\zeta = -i$. Numerical analysis shows that for $0 \leqslant \beta < 64.41°$ one more root is located on the circle in the third quadrant. This root corresponds to the leading critical point A on the plate. Depending on β, the remaining two roots are symmetrically located on the circle either in the third or in the first quadrant (i.e., only one of them is located in the flow field).

The pattern of the streamlines calculated near the plate for d = 0.134 is shown in Fig. 3. The ordinate y_1 of point A and the coordinates x_2 and y_2 of the critical point C in the stream are given in Table 1 for several values of d.

The dipole splitting is the most characteristic feature of the flow pattern outside the dipole zone. A part of the dipole strength is used to generate the closed dipole zone (which now has only one point D in common with the plate). The remaining part is used to capture a certain amount of fluid $q = \Delta\psi$ directly from the free stream and to eject exactly the same amount into the stream on the other side (see Fig. 3, where the filaments of the stream are partially shaded and the presence of splitting represents bifurcation of the dipole). The relation between the captured flow rate Δψ and the location of the dipole d is shown in Fig. 4. Note that the transverse extent of the filaments captured from the free stream determines the rate of the flow separated from the plate.

The above quantitative data are related to the case of flow past a plate for α = 90°. However, the presence of the sub- and supercritical regimes and, consequently, the point of bifurcation is also observed for small angles of incidence where the assumption that the flow past the leading edge does not separate can be approximately realized. To confirm the above, the distance from the trailing edge to the dipole $\bar{d}$ and to the critical point $\bar{s}$ in the limiting case versus α are shown in Fig. 5; both are related to the same quantities for the case of $\alpha = 90°$. Consequently, even for $\alpha \neq 90°$, the pattern of the discrete formation of the first (initial) vortex for the developing circulatory flow in an inviscid, incompressible fluid is retained.

However, the formation of the second vortex at the trailing edge of the plate cannot be described by employing only the dipole model. In order to obtain the correct pattern of the flow near the trailing edge after bifurcation occurs, it is necessary to realize that the amount of the fluid separated from the plate has already been acting on the flow as a vortex with circulation Γ_1. This strongly complicates the description of the flow. However, if one assumes that the second vortex will be formed only after the first one moves sufficiently far downstream, then the dipole model allows one to obtain an estimate of the circulation for the second vortex. For the plate with $\alpha = 90°$, $\Gamma_2 = 0.1631\Gamma_\infty$.

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DISPLACEMENT OF THE FREE SURFACE OF A FLUID DURING FLOW OVER A CYLINDER

I. S. Dolina, S. A. Ermakov, and E. N. Pelinovskii

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As is well known, the problem concerning potential flow over a cylinder by a fluid with a free surface may be reduced to integral equations [1], the exact solution of which is not known. In practice, much attention has been given to an approximation due to Lamb [1] in which the cylinder is replaced by a hydrodynamic dipole and the problem is then solved in a linear setting. Our purpose here is to provide experimental verification for Lamb's approximation and further perfection of his theory (taking into account the nonpotential nature of the flow over the cylinder).

An experiment was conducted in a Plexiglas trough (150 × 50 × 18 cm) in which a cylinder of radius R = 1 cm was moved at various speeds U (from 20 to 80 cm/sec) in a horizontal direction. As a rule, the full depth of the fluid was several times the depth h of immersion of the cylinder, so that the influence of the trough bottom was insignificant; Froude number $Fr = U^2/gh$ did not exceed 3. The free surface profile was studied photographically with the help of a scaling grid marked on a side wall of the tank; in addition, a conductivity data unit was used for sufficiently small displacements (of order less than 1 mm).

The form of the surface above the cylinder depends essentially on Fr; its form for Fr = 2 is shown in Fig. 1. When Fr > 1 a bulge forms above the cylinder, its maximum corresponding to the center coordinate of the cylinder; when Fr is decreased (Fr < 1.2-1.3) the bulge diminishes and is displaced forward. For small Fr (Fr = 0.3-0.5) bulges above the body are generally not observed; the water level goes down smoothly, going into a depression behind the cylinder. In the region behind the cylinder one observes a rapidly diminishing (of at most 1-2 periods) surface wave propagating with velocity U.

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